

Gravitational quantum effects in light of BICEP2 results

Tao Zhu* and Anzhong Wang†‡

GCAP-CASPER, Physics Department, Baylor University, Waco, TX 76798-7316, USA

Institute for Advanced Physics & Mathematics, Zhejiang University of Technology, Hangzhou, 310032, China

(Dated: April 3, 2014)

Recently BICEP2 found that the vanishing of the tensor-to-scalar ratio r is excluded at 7σ level, and its most likely value is $r = 0.2^{+0.07}_{-0.05}$ at 1σ level. This immediately causes a tension with the Planck constraint $r < 0.11$. In addition, it also implies that the inflaton (in single field slow-roll inflation) experienced a Planck excursion during inflation $\Delta\phi/M_{Pl} \geq \mathcal{O}(1)$, whereby the effective theory of inflation becomes questionable. In this brief report, we show that the inflationary paradigm is still robust, even after the quantum effects are taken into account. Moreover, these effects also help to relax the tension on the different values of r given by BICEP2 and Planck.

I. INTRODUCTION

The inflationary scenario provides a framework for solving the problems of the standard big bang cosmology and, most importantly, provides a causal mechanism for generating structure in the universe and the spectrum of cosmic microwave background (CMB) anisotropies. In this picture, primordial density and gravitational wave fluctuations are created from quantum fluctuations during the inflation process. The former, which generates observational effects in temperature anisotropies in CMB, has been conformed by observations with spectacular precision [1, 2]. For the primordial gravitational wave fluctuations, which generates the unique primordial B-mode polarization of CMB, has now been observed by the ground based BICEP2 experiment [3]. The results of the experiment shows for the first time a non-zero value of the tensor-to-scalar ratio r at 7σ C.L., and estimated

$$r = 0.2^{+0.07}_{-0.05}, \quad (1)$$

at 1σ C.L. If it is confirmed, it will represent one of the most important discoveries in the field, and provides us a new diagnostic tools to probe the models of early universe, especially the models in the framework of quantum gravity.

However, the BICEP2 results are in tension with the Planck constraint $r < 0.11$ [2]. As pointed out by BICEP2 group [3], this tension may be alleviated if a large negative running of scale spectral index is allowed. However, such a large negative running can not be produced in the framework of single field slow-roll inflation models. As the single-field slow-roll inflation has already been shown to be favored by the Planck data, it is very desirable to find some mechanisms to relieve this tension.

More important, a large r , such that given by Eq.(1), also implies that the effective field theory of inflation is problematic. This is based on the analysis of the Lyth

bound [4], which states that in the slow-roll approximations, the change of the inflationary field ϕ during inflation is given by

$$\frac{\Delta\phi}{M_{Pl}} \simeq \sqrt{\frac{r}{8}} \Delta N, \quad (2)$$

where M_{Pl} denotes the Planck mass, and ΔN the number of e-folds corresponding to when the observed scales in the CMB leave the inflationary horizon. For $2 \lesssim l \lesssim 100$, we have $\Delta N \simeq 3.9$, so that the corresponding field variation is

$$\frac{\Delta\phi}{M_{Pl}} \simeq 0.39 \sqrt{\frac{r}{0.07}}. \quad (3)$$

Obviously, $\Delta\phi$ exceeds the Planck scale if r is of the order of Eq.(1). As a result, the effective field theory of inflation with a potential $V(\phi)$, which consists of a derivative expansion of operators suppressed by Planck scale, becomes questionable.

In this brief report, we take the point of view that r is indeed large and of the order of Eq.(1), so that the gravitational quantum effects are important and necessary to be taken into account during the epoch of inflation. Once these effects are taken into account, we shall show that the inflation paradigm is still very robust, and almost scale-invariant can be easily produced. In addition, these effects also help to relax the tension between the values of r given, respectively, by Planck and BICEP2.

II. TRANS-PLANCKIAN EFFECTS

While quantum gravity has not been properly formulated, gravitational quantum effects on inflation have been studied by various methods [5–7]. One of them is to assume that the dispersion relations of scalar and tensor perturbations during inflation become nonlinear [8–11], similar to the study of Hawking radiation in black hole physics [12]. Remarkably, such relations can be realized naturally in the framework of Horava-Lifshitz quantum gravity [13–18], which is power-counting renormalizable by construction [19].

†Corresponding author

*Electronic address: tao.zhu@baylor.edu

‡Electronic address: anzhong.wang@baylor.edu

To show our above claim, let us first begin with the conventional linear dispersion relation $\omega_k^2(\eta) = k^2$, with which the inflationary perturbations $\mu_k(\eta)$ (scale or tensor) obey the equation

$$\mu_k''(\eta) + \left(\omega_k^2(\eta) - \frac{z''}{z} \right) \mu_k(\eta) = 0, \quad (4)$$

where a prime represents the differentiation with respect to conformal time η , $z(\eta)$ depends on the background and the types of perturbations, scalar or tensor, and usually is proportional to the cosmological scale factor $a(\eta)$.

After quantum effects are taken into account, the inflationary mode functions still satisfy the above equation but with a nonlinear dispersion relation [7, 13–18],

$$\omega_k^2(\eta) = k^2 \left[1 - \hat{b}_1 \left(\frac{k}{aM_*} \right)^2 + \hat{b}_2 \left(\frac{k}{aM_*} \right)^4 \right], \quad (5)$$

where M_* is the relevant energy scale of trans-Planckian physics, k is the comoving wavenumber of the mode, \hat{b}_1 and \hat{b}_2 are dimensionless constants, and in order to get a health UV limit, one requires $\hat{b}_2 > 0$.

To proceed, it is more convenient to introduce the dimensionless variable $y = -k\eta$, and write Eq.(4) in the form [20, 21]

$$\frac{d^2 \mu_k(y)}{dy^2} = [g(y) + q(y)] \mu_k(y), \quad (6)$$

where the convergence of the solutions requires the following choice of the functions $g(y)$ and $q(y)$ [22],

$$q(y) = -\frac{1}{4y^2},$$

$$g(y) = \frac{\nu^2}{y^2} - 1 + b_1 \epsilon_*^2 y^2 - b_2 \epsilon_*^4 y^4, \quad (7)$$

where $\epsilon_*^2 = H^2/M_*^2$, H is the Hubble parameter. Note that in the above we have set $z''/z \equiv [\nu^2(\eta) - 1/4]/\eta^2$ and $a \simeq -(1 - \epsilon + \mathcal{O}(\epsilon^2))/(\eta H)$, here the slow roll parameter ϵ has been absorbed in parameters $b_1 \equiv \hat{b}_1(1 + 2\epsilon + \mathcal{O}(\epsilon^2))$ and $b_2 \equiv \hat{b}_2(1 + 4\epsilon + \mathcal{O}(\epsilon^2))$.

To solve Eq.(6), we adopt the uniform asymptotic approximation developed recently in [22, 23]. We first note that the function $g(y)$ could have three physically different cases, as illustrated in Fig.1, depending on the number of zeros or turning points in terms of the terminology given in [20]. The corresponding roots will be denoted by y_0 , y_1 and y_2 , where y_0 is always real, while y_1 and y_2 can be both real, or both complex, or coalesce into one double root. As have been shown in [22], the approximate solutions of mode functions depend on the properties of these turning points.

With the approximate solutions for both scalar and tensor perturbations (Note that for both scalar and tensor modes, we have imposed the initial state as the

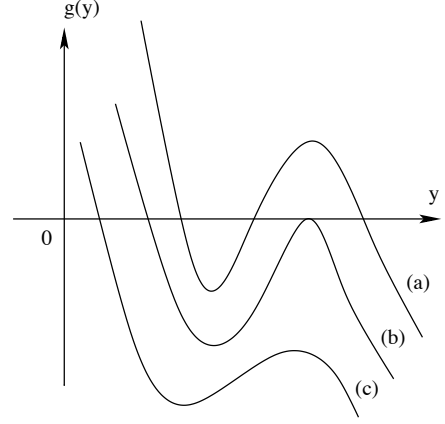


FIG. 1: The function $g(y)$ defined by Eq. (7). (a) Three different single real roots of the equation $g(y) = 0$. (b) One single and one double real root. (c) One single real root.

Bunch-Davies vacuum, for details, see [22]), one can compute the corresponding power spectra, which read,

$$\Delta^2(k) \equiv \frac{k^3}{2\pi} \left| \frac{\mu_k(y)}{z} \right|^2$$

$$= \frac{k^2 y}{4\pi^2 z^2 \nu} \mathcal{A} \exp \left(2 \int_y^{y_0} \sqrt{g(\tilde{y})} d\tilde{y} \right), \quad (8)$$

where the factor \mathcal{A} represents the trans-Planckian effects from the nonlinear terms in dispersion relation (5), could be amplified by the non-adiabatic evolution of inflationary perturbations, and is given by

$$\mathcal{A} \equiv (1 + 2e^{\pi\xi_0^2} + 2e^{\pi\xi_0^2/2} \sqrt{1 + e^{\pi\xi_0^2}} \cos 2\mathfrak{B}), \quad (9)$$

with

$$\xi_0^2 \equiv \frac{2}{\pi} \int_{y_1}^{y_2} \sqrt{g(y)} dy, \quad (10)$$

$$\mathfrak{B} \equiv \int_{y_0}^{\text{Re}(y_1)} \sqrt{-g(y)} dy + \phi(\xi_0^2/2), \quad (11)$$

where $\phi(x) \equiv x/2 - (x/4) \ln x^2 + \text{ph}\Gamma(ix + 1/2)/2$ with $\text{ph}\Gamma(ix + 1/2)/2$ is zero when $x = 0$, and is determined by continuity otherwise. In the above the quantity ξ_0^2 depends on the properties of the turning points. ξ_0^2 could be positive, zero, and negative, corresponding to the facts that $g(y)$ has three real turning points, one single turning point and one double turning point, and one single and two complex turning points, respectively. It is obviously that when ξ_0^2 is positively large, the power spectra is amplified due to the non-adiabatic evolution, and when ξ_0^2 is negatively large, it means the adiabatic condition for the equation of motion is satisfied, and for this case the modified factor \mathcal{A} is order of 1.

In order to study the effects of the nonlinear terms appearing in dispersion relations, represented by the b_1 and b_2 terms, one needs to perform the integral in Eq.(8)

explicitly. For the scalar modes we identify the curvature fluctuation as $\mathcal{R}^2 = \mu_k^2/z_s^2$ with $z_s = \sqrt{2\epsilon}a$, and $h^2 = 8\mu_k^2/z_t^2$ for the tensor modes with $z_t = a$. Thus the general expression of tensor-to-scalar ratio can be written as

$$r = \frac{8\Delta_t^2(k)}{\Delta_s^2(k)} = r_{\text{GR}} \left(\frac{\mathcal{A}_t}{\mathcal{A}_s} \sigma_k \right), \quad (12)$$

where $r_{\text{GR}} = 16\epsilon$ represents the tensor-to-scalar ratio predicted in slow-roll inflation models with a linear dispersion relation $\omega_k^2 = k^2$ obtained in general relativity, and

$$\sigma_k \equiv \exp \left(2 \int_{y_0}^{y_0^t} \sqrt{g_t(x)} dx - 2 \int_y^{y_0^s} \sqrt{g_s(x)} dx \right), \quad (13)$$

where y_0^t , $g_t(y)$ are associated quantities for tensor modes, while y_0^s , $g_s(y)$ for scalar modes¹. Comparing r with that in GR, one see that the factor $\mathcal{A}_t\sigma_k/\mathcal{A}_s$ represents the trans-Planckian effects on the tensor-to-scalar ratio. To reconcile the tension between BICEP2 and Planck, it is required that

$$\frac{\mathcal{A}_t}{\mathcal{A}_s} \sigma_k \gtrsim \mathcal{O}(2). \quad (14)$$

In the following we will show that this condition can be easily achieved by properly choosing the parameters of the nonlinear terms in the dispersion relations.

Let us first take a look at Eq.(14), from which we can see that the above condition can be realized by two possible ways. The first mechanism is to raise the factor $\mathcal{A}_t/\mathcal{A}_s$ by incorporating the non-adiabatic effects, while assuming $\sigma_k \simeq 1$. Note that the later condition can be easily realized by assuming that ϵ_*^2 is very small. However, as discussed in [24–26], once the non-adiabatic evolution is involved, a curial question is whether the back-reaction of the non-adiabatic modes is small enough to allow inflation to proceed. According to the analysis given in [26], one has to constrain the ratio of the modification of scalar and tensor power spectra to the limits,

$$0.69^2 < \frac{|\alpha + \beta|^2}{|\tilde{\alpha} + \tilde{\beta}|^2} < 1.44^2, \quad (15)$$

in order for the inflation to last for an enough long time, where β and $\tilde{\beta}$ represent, respectively, the Bogoliubov coefficients of the scalar and tensor modes, generated by the non-adiabatic evolution of the inflation. Here it is easy to identify that $\mathcal{A}_t = |\alpha + \beta|^2$ and $\mathcal{A}_s = |\tilde{\alpha} + \tilde{\beta}|^2$, thus one requires

$$0.48 < \frac{\mathcal{A}_t}{\mathcal{A}_s} < 2.07. \quad (16)$$

¹ It should be noted that the constants \hat{b}_1 and \hat{b}_2 appearing in the dispersion relation (5) are independent and different for scalar and tensor perturbations, as shown in [13–18].

Clearly, for $\mathcal{A}_t/\mathcal{A}_s \simeq \mathcal{O}(2)$, the requirement (14) is fulfilled. Even though this provides a possible way to reconcile the tension between Planck and BICEP2, it should be noted that the back-reaction of the non-adiabatic evolution is very difficult to calculate, and often depends on the specific theories, tedious and complicated analysis.

The second possibility is to raise the factor σ_k by properly choosing the parameters of nonlinear terms in the dispersion relations. For the sake of the simplification, in this case let us assume that the adiabatic condition is satisfied during inflation for both the scalar and tensor modes, thus we have $\mathcal{A}_s \simeq 1 \simeq \mathcal{A}_t$. In order to evaluate σ_k , one has to perform the integrals in Eq.(13) explicitly. However, these integrals, which involve nonlinear dispersion relations, are very difficult to compute. In order to see the effects of the nonlinear terms clearly, we can make some additional assumptions in the integrals. Let us first assume that the energy scale of inflation is less than the trans-Planckian scale M_* , i.e., $\epsilon_*^2 = H^2/M_*^2 < \mathcal{O}(1)$, thus the integrand of the integral in (13) can be expanded in terms of ϵ_* . Up to order of $\mathcal{O}(\epsilon_*^4)$ and after some simple but tedious calculations we find that

$$\begin{aligned} \sigma_k \simeq & \left(\frac{y_0^t}{\nu_t} \right)^{2\nu_t} \left(\frac{\nu_s}{y_0^s} \right)^{2\nu_s} \\ & \times \exp \left(-\frac{1}{3} b_1^t \epsilon_*^2 y_0^{t3} - \frac{b_1^{t2} y_0^{t5} \epsilon_*^4}{5} + \frac{7b_2^t y_0^{t5} \epsilon_*^4}{15} \right) \\ & \times \exp \left(\frac{1}{3} b_1^s \epsilon_*^2 y_0^{s3} + \frac{b_1^{s2} y_0^{s5} \epsilon_*^4}{5} - \frac{7b_2^s y_0^{s5} \epsilon_*^4}{15} \right), \quad (17) \end{aligned}$$

with

$$\begin{aligned} y_0^t & \simeq \nu_t + \frac{b_1^t \nu_t^3}{2} \epsilon_*^2 + \frac{1}{8} (7b_1^{t2} - 4b_2^t) \nu_t^5 \epsilon_*^4 + \mathcal{O}(\epsilon_*^6), \\ y_0^s & \simeq \nu_s + \frac{b_1^s \nu_s^3}{2} \epsilon_*^2 + \frac{1}{8} (7b_1^{s2} - 4b_2^s) \nu_s^5 \epsilon_*^4 + \mathcal{O}(\epsilon_*^6). \quad (18) \end{aligned}$$

In the slow-roll approximations, we have $\nu_t \simeq \frac{3}{2} \simeq \nu_s$. Then, up to the order of ϵ_*^2 we find that

$$\sigma_k \simeq \left(\frac{1 + 9b_1^t \epsilon_*^2/8}{1 + 9b_1^s \epsilon_*^2/8} \right)^3 \exp \left[\frac{9}{8} (b_1^s - b_1^t) \epsilon_*^2 \right]. \quad (19)$$

In order to have $\sigma_k \simeq \mathcal{O}(2)$, one can either raise the tensor spectrum or suppress the scalar spectrum. To raise the tensor spectrum, one needs to choose $b_1^t > 0$ and $b_1^t \epsilon_*^2 \simeq \mathcal{O}(0.5)$. However, from Eq.(7) one sees that, when these conditions are satisfied, $g(y)$ shall generically have three real turning points, which means that the adiabatic condition is violated and \mathcal{A}_t shall exceed the bound given in Eq.(15). Another way to get $\sigma_k \simeq \mathcal{O}(2)$ is to suppress the scalar spectrum. In this case, if one chooses $b_1^s < 0$ and $b_1^s \epsilon_*^2 \simeq \mathcal{O}(0.5)$, σ_k will raise from 1 to $\mathcal{O}(2)$. For examples, from Eq.(19) we find that $\sigma_k \simeq 1.5$ for $9b_1^s/8\epsilon_*^2 \simeq -0.18$, and $\sigma_k \simeq 2$ for $9b_1^s/8 \simeq -0.28$, all with $b_1^t \epsilon_*^2 \ll 1$.

III. CONCLUSIONS

In this brief report, we have taken the point of view that the tensor-to-scalar ratio r is big, as found by BICEP2, and that the trans-Planckian effects become indeed important and need to be taken into account during the epoch of inflation, as indicated by the Lyth bound (2). Then, we have shown that, even after these effects are taken into account, almost scale-invariant perturbations can still be easily obtained [13, 16–18], and the inflation paradigm is robust. Moreover, these effects also helps to relax the tension between the values of r given,

respectively, by Planck BICEP2, whereby the two problems mentioned in Introduction are resolved.

Acknowledgements

This work is supported in part by DOE Grant, DE-FG02-10ER41692 (AW); Ciência Sem Fronteiras, No. 004/2013 - DRI/CAPES (AW); NSFC No. 11375153 (AW), No. 11173021 (AW), No. 11047008 (TZ), No. 11105120 (TZ), and No. 11205133 (TZ).

-
- [1] E. Komatsu *et al.* (WMAP Collaboration), *Astrophys. J. Suppl. Ser.* **192**, 18 (2011); D. Larson *et al.* (WMAP Collaboration), *Astrophys. J. Suppl. Ser.* **192**, 16 (2011).
 - [2] P.A.R. Ade *et al.* (PLANCK Collaboration), arXiv: 1303.5082.
 - [3] P.A.R. Ade *et al.* (BICEP2 Collaboration), *BICEP2 I: Detection Of B-mode Polarization at Degree Angular Scales*, arXiv: 1403.3985.
 - [4] D.H. Lyth, *Phys. Rev. Lett.* **78**, 1861 (1997).
 - [5] A. Ashtekar, arXiv:1303.4989.
 - [6] C.P. Burgess, M. Cicoli, F. Quevedo, arXiv:1306.3512.
 - [7] R.H. Brandenberger and J. Martin, *Class. Quantum Grav.* **30** (2013) 113001.
 - [8] R.H. Brandenberger, arXiv:hep-th/9910410.
 - [9] J. Martin and R.H. Brandenberger, *Phys. Rev. D* **63**, 123501 (2001); *Phys. Rev. D* **65**, 103514 (2002); *Phys. Rev. D* **68**, 063513 (2003).
 - [10] J.C. Niemeyer and R. Parentani, *Phys. Rev. D* **64**, 101301 (R) (2001).
 - [11] J. Martin and C. Ringeval, *Phys. Rev. D* **69**, 083515 (2004).
 - [12] W. Unruh, *Phys. Rev. D* **51**, 2827 (1995); R. Brout, S. Massar, R. Parentani, and P. Spindel, *ibid.* **D52**, 4559 (1995); N. Hambli and C. Burgess, *ibid.*, **D53**, 5717 (1996); S. Corley and T. Jacobson, *ibid.*, **D54**, 1568 (1996); S. Corley, *ibid.*, **D57**, 6280 (1998).
 - [13] A. Wang and R. Maartens, *Phys. Rev. D* **81**, 024009 (2010); A. Wang, D. Wands, and R. Maartens, *JCAP*, **03**, 013 (2010); J.O. Gong, S. Koh, and M. Sasaki, *Phys. Rev. D* **81**, 084053 (2010).
 - [14] T. Kobayashi, Y. Urakawa, and M. Yamaguchi, *JCAP*, **04**, 025 (2010); E.G.M. Ferreira and R. Brandenberger, *Phys. Rev. D* **86**, 043514 (2012).
 - [15] T. Takahashi and J. Soda, *Phys. Rev. Lett.* **102**, 231301 (2009); A. Wang, Q. Wu, W. Zhao, and T. Zhu, *Phys. Rev. D* **87**, 103512 (2013).
 - [16] A. Wang, *Phys. Rev. D* **82**, 124063 (2010).
 - [17] Y. Huang, A. Wang, and Q. Wu, *JCAP*, **10**, 010 (2012).
 - [18] T. Zhu, Y. Huang, and A. Wang, *JHEP*, **01**, 138 (2013).
 - [19] P. Hořava, *Phys. Rev. D* **79**, 084008 (2009).
 - [20] F.W.J. Olver, *Asymptotics and Special functions*, (AKP Classics, Wellesley, MA 1997).
 - [21] F.W.J. Olver, *Second-order linear differential equations with two turning points*, *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 278, 137 (1975).
 - [22] T. Zhu, A. Wang, G. Cleaver, K. Kirsten, and Q. Sheng, *Phys. Rev. D* **89**, 043507 (2014); arXiv:1308.1104.
 - [23] S. Habib, K. Heitmann, G. Jungman, and C. Molina-Paris, *Phys. Rev. Lett* **89**, 281301 (2002); S. Habib, A. Heinen, K. Heitmann, G. Jungman, and C. Molina-Paris, *Phys. Rev. D* **70**, 083507 (2004).
 - [24] H. Collins, R. Holman, and T. Vardanyan, arXiv:1403.4592.
 - [25] A. Ashoorioon, K. Dimopoulos, M. M. Sheikh-Jabbari, G. Shiu, arXiv:1403.6099; *JCAP* **02**, 025 (2014).
 - [26] A. Aravind, D. Lorshbough, and S. Paban, arXiv: 1403.6216.